Network Planning using Geomorphology

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ABSTRACT

Connecting several locations, for example towns by roads, is a typical network planning problem. A first approach would be to connect pairs of nodes (network language term for location) with a straight edge. The result is a connected network not considering the total network length and the average trip length. The construction of new junctions, so called Steiner Points, for reducing the total network length and thereby the global costs is more or less known. On the other hand slopes, obstacles or other unsteady costs can not be taken into account during construction. Therefore an independent and homogeneous representation for costs and network planning is necessary.

Our approach distinguishes between topographical and additional costs, for example nature reserves and restricted areas. The proposed representation of the topography in a digital terrain model (DTM) is a constrained delaunay triangulated irregular network (TIN). The initial problem of connecting different locations is at the first sight reduced to the shortest path problem, taking the extra attributes into account. But these algorithms are not suitable to connect more than two locations.

In this paper we will show how to use the information of a slightly extended Dijkstra Shortest-path algorithm to classify the nodes of the TIN and identify candidates for Steiner Points. Assuming that each geomorphologically closed area can be represented by one Steiner Point, the presented algorithm leads to an optimal network. Steiner Points, passes and the shortest paths between the latter constitute the basis for a network, which provides cheapest connections with regard to topographical costs.

To avoid exhaustive searching, geomorphology of the DTM is used as a powerful heuristic. It leads to a partitioning of the space and as a result to a divide- and conquer strategy.

Categories and Subject Descriptors

G.2.1 **[Discrete Mathematics]**: Combinatorics – *combinatorial algorithms*; G.2.2 **[Discrete Mathematics]**: Combinatorics – n*etwork problems*; H.2.8 **[Database Management]**: Database Applications – *spatial databases and GIS*; I.2.8 **[Artificial Intelligence]**: Problem Solving, Control Methods, and Search – h*euristic methods*.

General Terms

Algorithms, Design, Human Factors.

Keywords

GIS, Network Planning, Digital Terrain Model (DTM), Triangulated Irregular Network (TIN), Geomorphology, Water Flow, Road Planning, Steiner Points.

1. INTRODUCTION

This paper deals with network planning particularly about "efficient" connections of multiple locations taking different kinds of costs into account. The focus of the presented approach is on road network extension, but it is easy to apply to any network planning task with similar cost modeling, like railroads or supply networks. The trick is a skilful representation of costs and exploiting knowledge of the earth science. The aim of the network extension is to connect multiple locations with least construction and travel cost.

The topography is represented by a TIN (Triangulated Irregular Network) [9], [8] with extra attributes at edges and nodes for additional costs such as nature reserves or restricted areas.

If the TIN is interpreted as a network, shortest-path algorithms (Dijkstra) can be used to find the cheapest shortest path between two points taking topographical costs into account. The network is thus just a suitable discrete representation of the terrain. The problem here however is not to find shortest paths but to design an optimal network. The shortest paths in the TIN identified by the shortest-path algorithm are however good candidates for connections of the desired network. Since the problem of the network is to connect n locations rather than two we additionally need the concept of Steiner Points bearing the crucial intersections of the network.

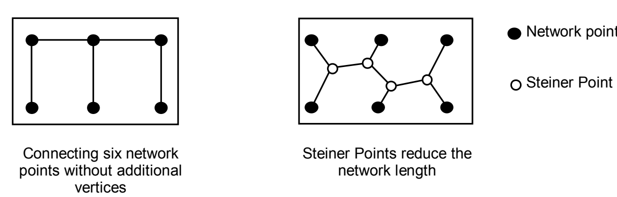

Figure 1. Steiner Points

Finding optimal Steiner Points is known to be NP-hard, i.e. known algorithms which guarantee optimal solutions are exponential in the worst case [6], [7]. Thus powerful heuristics are needed which additionally support prioritization and weighting of locations.

Geomorphology provides a basis for a powerful heuristic by a natural partitioning of the terrain. Such a partition is given by the derivation of basins and watersheds, where the passes are the main connectors, and basins are separated by watersheds. This leads to a partitioning of the surface and so to a divide-and-conquer-strategy for the network planning. Therefore the surface is represented by a constrained Delaunay TIN [3], [9].

We use the term geomorphology in a rather restricting sense focusing on water flow under the additional assumption that the surface is impermeable.

To derive the geomorphology for each triangle the water flow will be calculated, thus the water drainage of the whole TIN and consequently basins (water catchment areas) can be derived (divide-step). Within these basins the connection costs are moderate. Interconnection between them will be done through passes, the cheapest point to cross the dividing ridge between basins. This leads to an algorithm finding "local" connections in basins including Steiner Points and to connect the basins also with Steiner Points in a further step (conquer-step) on a higher, more abstract level. The method for connecting locations and basins is an extended Dijkstra shortest path algorithm. During the generation of paths the algorithm considers length, slope, winding and priority of nodes/locations to connect with a given weighting.

2. DIGITAL TERRAIN MODEL (DTM)

To achieve realistic results the modeling of costs has to take several concerns into account. Mathematically the costs can be divided into two major parts, the local and global costs.

Global costs are properties of the graph itself and not reducible to costs of simple edges. Seperability and connectivity being the most prominent examples. Our focus is on local costs which can be represented as attributes of simple edges. Local costs can be divided into costs which are caused by the topography (shape of the surface) and additional costs caused by different concerns such as nature reserves, political restrictions etc. Since additional costs cannot be integrated into the digital terrain they will be stored in extra attributes of the TIN which represents the terrain. Other additional costs like winding, priorities of connections between certain locations and so on will also be taken into account (see section 4. Shortest Paths). All modeled costs will additionally be weighted by parameters.

Another approach could be to split the costs in a construction and a running part. This would help to ascertain the economic costs but would not lead to a model. With the proposed model these costs can be determined after the generation of paths but also indirectly integrated in the mentioned costs.

Topography can be represented by a digital terrain model (DTM). This data model is based on a Delaunay triangulation. However, since ordinary Delaunay triangulation only represents the relief, it does not consider additional obstacles. Constrained Delaunay triangulation, which is used for the representation of breaklines, can be applied and adopted in order to tightly integrate the locations to connect, selective and local costs into the triangulation of the terrain.

The path-generation process for connecting the given locations will be applied to this triangulation and the result will be a path using its edges as explained later in this paper. During this construction, the additional costs, stored in extra attributes at the nodes and edges of the triangulation, can be considered.

3. GEOMORPHOLOGY

The proposed approach is to use the geomorphologic structure of the terrain [10], [4], [11] as a base heuristic (see Figure 6). On the one hand the geomorphology partitions the space (TIN) by the identification of water catchment areas (basins). This is the divide step, which reduces the running time. On the other side the geomorphology provides information about connecting these basins with modest cost through passes (conquer step). To obtain these information about the geomorphology, the water flow of the terrain must be located.

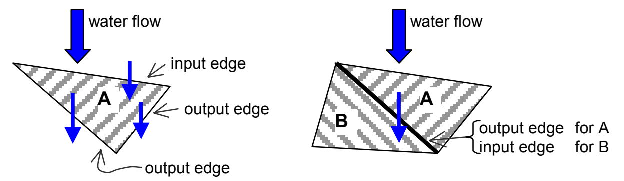

Figure 2. Triangle edges and water flow

3.1 Water Flow

The advantage of the introduced representation by a TIN is the unambiguous derivation of the water flow from each triangle. For simplifications each triangle should not be plain. Here the edges and its water flow must be observed closer. An edge can be an input or an output edge to its affiliated triangle related to the water flow or rather to its position in the space. It is obvious that an edge has in general two opposite values for its affiliated triangles (see Figure 2).

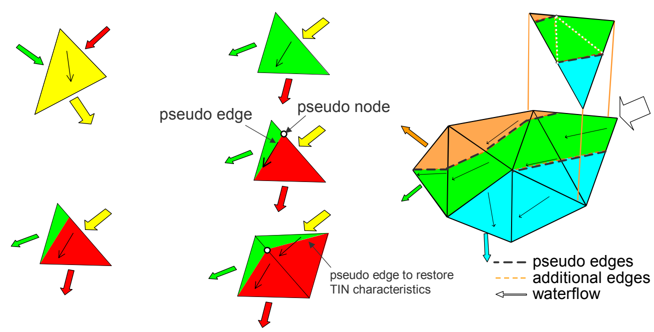

Figure 3. Water flow (left), derivation of pseudo edges (3 middle figures) and propagation of pseudo-edges (right)

A triangle can have two input and one output edge or one input and two output edges (see Figure 2). Triangles with two output edges are problematic because these ones cannot be unambiguously assigned to one water catchment area (see Figure 3). This forces a splitting of the triangle by a so called pseudo-edge (see Figure 3). The pseudo-edge is parallel to the water flow and can be interpreted as an input edge for the both new triangles. This leads to a new node, so called pseudo node, at the former input edge. To maintain the TIN characteristic the triangle incident to the splitted edge must also be divided. The new splitting edge in the incident triangle will start at the pseudo node and end at the opposite triangle node, the node that is not adjacent to the pseudo node (see Figure 3). The initial triangle is completed, the second one was treated to restore the TIN characteristics without considering the water flow – input and output edges. The water flow of the both new triangles emerged from the second triangle must be checked. This leads to a recursive process. For determinism all triangles with two output edges must be put in a stack, sorted by the altitude of the lowest node of the assigned triangle. Due to the ascending work off of the triangles with two output edges the process is finite. A closer running time analysis is beyond the topic of this paper.

3.2 Basins

When water falls in a water catchment area, it cannot leave this area, ignoring a run over. Due to the water accumulation these areas are also called basins. Hollows, lowest points of a basin, are special points. To identify hollows all nodes of the network must be locally examined. The tested node is a hollow if all adjacent nodes have higher z-coordinates.

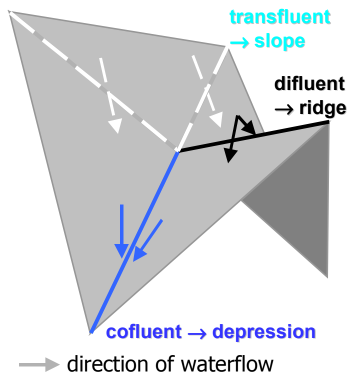

Figure 4. Water flow in drainage networks

Not each hollow represents an own basin. If there are multiple hollows in one basin the lowest hollow is the representative. Therefore the located hollows must be worked off starting with the lowest one not included in any other basin. Without this order the process is non-deterministic.

In the last chapter the water flow between triangles and its edges was described. Here the connection between water flow on edges and both incident triangles will be analyzed. If water flows from one triangle to a neighboring triangle above the separating edge (a slope), this edge is called transfluent (see Figure 4). The edge is an output edge for the first triangle and an input edge for the second triangle. A confluent edge is a depression; the edge is an output edge for both incident triangles. If the edge is an input edge for both triangles, the edge is a ridge, a diffluent edge.

The algorithm processes the ordered list of hollows starting from the lowest one. All triangles incident to the hollow, if not already assigned to a basin, are the starting point for the new basin. Recursively all adjacent not assigned triangles will be added to the basin if the dividing edge is not diffluent and the edge is an input edge for the basin up to now. Attention must be turned to the derivation of the basin border, the watersheds, within the same run (running time) and to intercepting triangles at the border of the research area. These triangles belong to basins outside of the research area and are not part of any "normal" derived basins.

These preparations will partition the space, i.e. this is the divide step. Before dealing with the path-generation, the geomorphology phase should be finished by the derivation of passes.

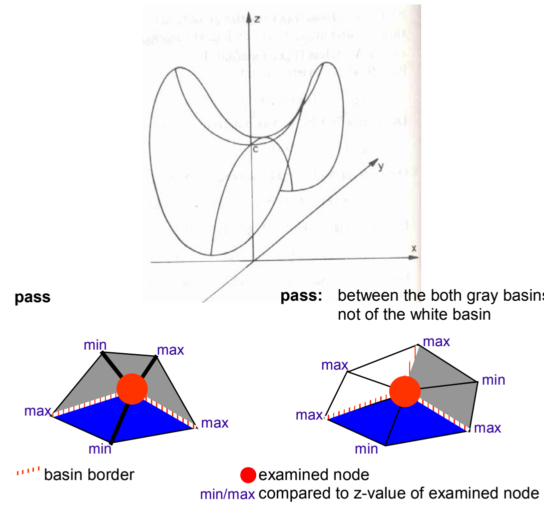

Figure 5. Mathematical definition of a pass (top); "normal" pass (left) and ambiguity of a pass (right)

3.3 Passes

In mathematical terms a pass is a saddle point (f´(x,y)=f´´(x,y)= 0, whereas (x,y) is neither a maximum nor a minimum) (see Figure 5). Not every saddle point, however, is a pass. Significance of passes comes from the fact that they connect different basins. Thus a pass is always located on a watershed.

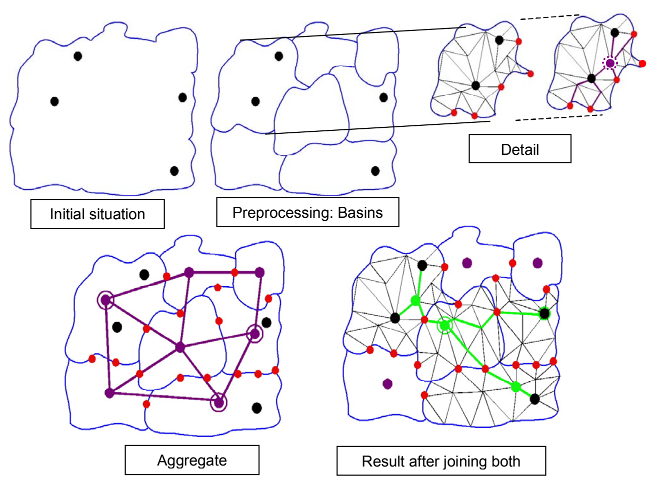

Figure 6. Locations to connect (top left); Derived basins (top middle); Detail Level (top right): Basin with passes (left of the both small figures) and shortest path with Steiner Point (right figure of the small ones); Basins with passes and aggregate network (bottom left); Detail and aggregate level with final shortest path (bottom right)

Adapted to the model three different kinds of passes can be extracted. The "normal" or "significant" pass lies on a border of basins, is a local minimum on this border and there exists a subsequence minimum–maximum–minimum–maximum to the adjacent nodes (see Figure 5). An "insignificant" pass is not lying on a border of basins and fulfills the remaining criteria. These passes are irrelevant. An "artificial" pass fulfills all the pass criteria except the subsequence on borders between two basins, to ensure that each neighborhood has at least one pass.

During the path-generation the "normal" and the "artificial" passes will be considered.

4. SHORTEST PATHS

The generation of paths is divided in two levels (see Figure 6). At the detail level locations and passes of each basin will be connected basin-internal including the construction of Steiner Points. Afterwards, in the aggregate level, a network will be derived, where each node represents one basin. An edge represents the neighborhood of basins, the incident nodes, if they have at least one common pass together. On this aggregate network a path-generation algorithm, similar to the detail level, will connect basins with locations including Steiner Points.

4.1 Detail Level

Basins and passes are derived during the geomorphology preprocessing. The initial locations to connect are integrated in the constrained Delaunay TIN as nodes and these nodes are marked as well as the pass-nodes. The detail level concentrates on basins. The further handling depends on the number of nodes to connect, the marked ones. In the case of one node (supposed to be a pass), there is nothing to do. If there are two nodes to connect the shortest path between these must be constructed considering given weightings. For the aggregate level a (pseudo) Steiner Point will be generated on the halfway. Otherwise an extended Dijkstra-Algorithm will be used.

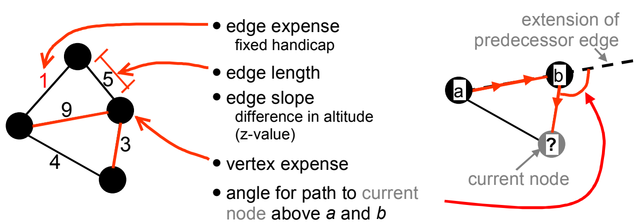

Figure 7. Local graph expenses

Dijkstra's well known algorithm [2], [1] solves the single-source shortest-path problem. It determines for each node of a network the lowest connection cost to a given one. The expenses must be all positive. The costs can be divided into two categories. Edge expense, edge length and edge slope (difference in altitude) can be (locally) attributed to each edge taking weightings into account. This is part of the classical Dijkstra shortest-path algorithm. In our context over and above the length of a path an important demand is smoothness, as little slopes as possible, and straightness, as little twisting as possible. Straightness is not a local but a semi-local property because it describes relations between consecutive edges in a path. Since Dijkstra is restricted to local costs attributed to single edges, an extension of this well-known algorithm is required.

The Dijkstra algorithm provides a data-structure supporting the shortest paths generated so far. In our context we augment this data-structure such that easy access to the angle with the predecessor edge will be supported. To derive the straightness for each pair of edges within the path the angle to its predecessor edge has to be calculated (see Figure 7).

In some cases, e.g. lakes, restricted areas etc, it is desirable to assign expenses to vertices rather than to edges. The integration of these costs is straight forward. The weighting for the parameters of slope, straightness, length etc is up to the theory of traffic planning and beyond the scope of this paper.

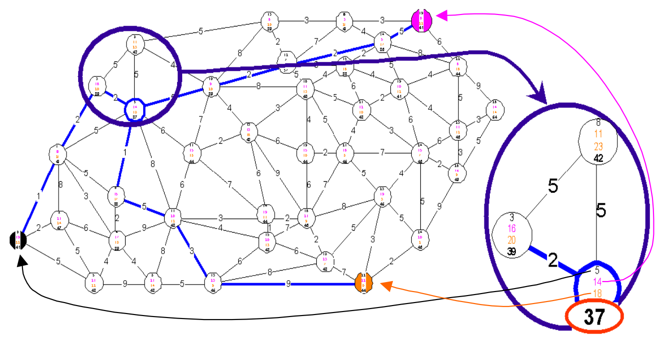

Figure 8. Nodes with expense list to each node to connect and derivation of a Steiner point in a basin

Our aim is to assess nodes in order to identify candidates for Steiner Points. Therefore we let the introduced algorithm run for each node to connect. The attribute for path-reconstruction and the cost of the cheapest path must therefore be stored in an array (see Figure 8). The array-index should correspond to the node to connect. The result is a list of least expense to each start-node at each node with its path-reconstruction link. The sum of the expense list expresses the cost from the specific node to all start-nodes. Consequently the node with the lowest cost-sum is the Steiner Point to the appropriated basin. The construction of the local final net in the basin is obvious. This procedure is quite running time intensive and beside the restriction to one Steiner Point the reason for partitioning the space.

4.2 Aggregate Level

In the aggregate level each basin will be represented by one node (see Figure 6 and Figure 9). Nodes of basins with locations to connect will be marked. On closer examination the similarity to the detail level is evident. The objective is to apply the extended Dijkstra-Algorithm of the previous chapter.

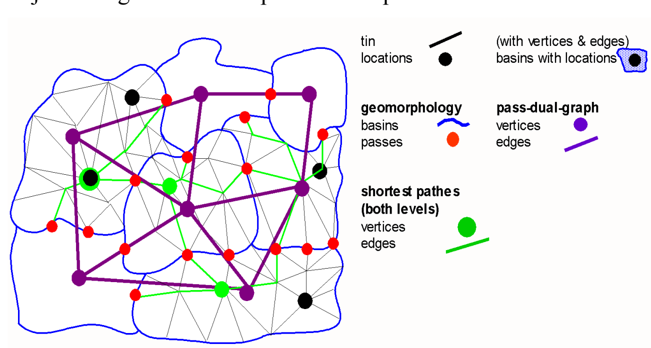

Figure 9. Partitioning of the study area in basins and the two-level model

An edge between two nodes respectively basins exists if these two basins have at least one common pass. The difficulty is to apply

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input:
• DTM in constrained Delaunay TIN representation (input TIN)
• additional costs as attributes of the TIN (cost-attributes)
• list of locations (nodes) to connect
• cost parameters

step 1: geomorphology
• derive water flow TIN from input TIN
• derive basins from water flow TIN
• derive passes from basins and water flow TIN

step 2: detail level
• for each basin do
     construct detail shortest-path with Steiner Point from water flow TIN with cost-attributes and
        passes and location considering cost parameters

step 3: aggregate level & graph
• aggregate-graph-node represents basin
• aggregate-graph-edge connects two nodes/basins if there exists a common pass
• construct aggregate shortest-path with Steiner Point on aggregate-graph with cost-attributes and
    connecting locations considering cost parameters

step 4: matching of both levels to the output shortest-path-network
• replace aggregate-graph-edges of the result of the aggregate shortest-path with Steiner Points by
    corresponding shortest-paths derived in the detail level

output:
• shortest-path-network (detail level with Steiner Points) connecting the locations
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Figure 10. Outline of the proposed algorithm

costs to this aggregated graph respectively to its edges. The trick is to use the retrieved Steiner Points of the basins during the detail level. These Steiner Points, in general located at the center, are the representative of the accompanying basins.

An edge of the aggregate graph describes the connection between its incident nodes respectively basins above at least one pass. This means the expense of this edge is the expense from the Steiner Point of one basin to the common pass and from there to the Steiner Point of the other basin. If there is more than one common pass, the one with least connection costs between the two Steiner Points is selected. This was the reason to introduce Steiner Points also to basins with less than three locations to connect.

From now on the method is equal to the detail level but using the new aggregate graph. The input for the extended Dijkstra-Algorithm are the nodes to connect. The additional local expenses are restricted to the derived edge costs. The node with the lowest total expense sum is a Steiner Point. So the net of the aggregate level is retrieved (used edges). The next step is the unification of both levels respectively networks.

The final network consists of all paths between two Steiner Points via a pass (Steiner Point – pass – Steiner Point) where the edge of the aggregate network represents this pass and the edge is used by the final aggregate network. Additionally all shortest paths on the detail network from the locations to connect to their accompanying Steiner Points of their basin are elements of the final network.

5. RUNNING TIME

The running time of deriving Steiner Points at the detail level is $O(v_b^2\, l_b)$ per basin, where v_b represents the number of vertices and l_b the number of locations, passes included, per basin. At the aggregate level the running time for deriving Steiner points is $O(b^2\, l_h)$, where b represents the number of basins and l_h the number of locations. The total running time is $O(b^2\, l_h + b\,(v_b^2\, l_b))$ plus the running time of the extraction of the geomorphology $O(v_t)$, where v_t is the total number of vertices. Thus the running time is $O(v_t^4)$ in the worst case.

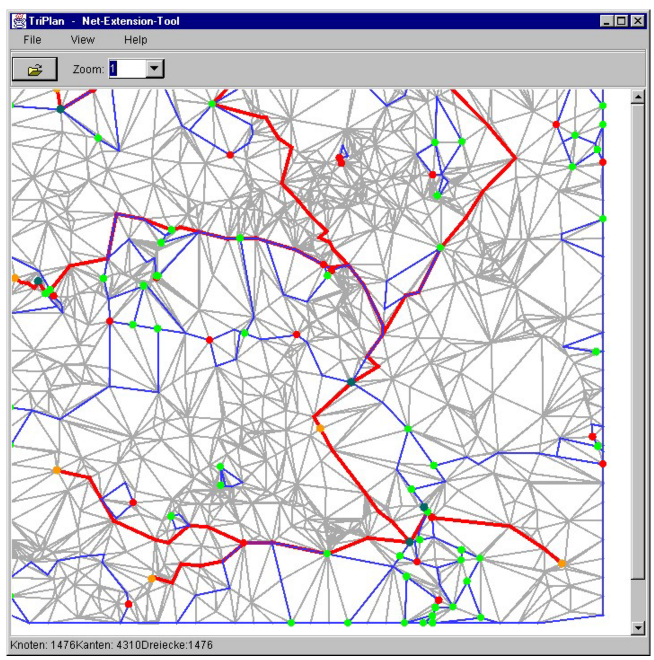

Figure 11. Screenshot of the prototype: Result of a test scenario

6. CONCLUSIONS

The paper introduces a new method for network planning including the derivation of additional junctions for reducing the total length and costs of networks. It is based on the exploitation of a DTM, a triangulated digital terrain model, that represents topography as well as additional costs in attributes. Global costs are derived during path generation by an extended Dijkstra-Shortest-Path-Algorithm. This adapted algorithm considers on the one hand the local cost, the topography of the DTM and the additional costs. On the other hand it sums up costs like smoothness and straightness. The evaluation of parameter weighting is up to the theory of traffic planning and beyond the scope of this paper. Another further development of the algorithm is the derivation of junctions. There are also concepts to derive more than one Steiner Point by this method. Due to the reduction of the complexity and the running time a heuristic is introduced. The heuristic presented here is based on geomorphology. It partitions the space in a natural way and so to reduce the problems to minor ones. This technique is known in computer science as divide-and-conquer. The conquer-step is to assemble the small networks to one homogeneous network. This step also exploits the geomorphology, the passes. An overview of the proposed algorithm is illustrated in Figure 10.

Originally this method was developed as a component of a research project on sustainable transport planning in Israel and Palestine (see Acknowledgement) [5]. The objective was to redevelop the exhausted road network in Palestine considering topographic, political and ecologic constraints. One of the results of the project was a social economic model which provides traffic demands for different future scenarios. In the evaluation phase of the algorithm described in this paper, these demands were successfully used as sample inputs (see Figure 11).

The presented method is implemented in a prototype using Java. It has interfaces to commercial GIS, such as ArcInfo.

7. ACKNOWLEDGMENTS

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